Journal of Sound and Vibration (2002) **257**(5), 815–837
doi:10.1006/jsvi.5062, available online at http://www.idealibrary.com on IDEAL®

EFFECTIVE VIBRATION ANALYSIS OF IC ENGINES USING CYCLOSTATIONARITY. PART I—A METHODOLOGY FOR CONDITION MONITORING

J. Antoni, J. Daniere and F. Guillet

Laboratory for the Analysis of Signals and Processes in the Industry, 20 *Avenue de Paris,* 42334 *Roanne, France. E-mail*: *jerome.antoni@hds.utc.fr*

(*Received* 30 *May* 2001, *and in final form* 17 *December* 2001)

Condition monitoring of IC engines through the analysis of their vibrations has been recognized to be a difficult issue, essentially because of the complexity of the signals that are involved. This paper introduces an original approach which explicitly takes into account the non-stationary nature of the vibration signals. The key idea is to rephrase the issue into the framework of cyclostationary processes, which is perfectly suited for describing physical phenomena generated on a cyclic basis. A general methodology is introduced that is based on angular sampling and cyclic signal processing. It serves as a strong basis for designing *ad hoc* diagnostic indicators.

1. INTRODUCTION

In the past decades, many successful efforts have been made to use vibration analysis as a means for condition monitoring of rotating machinery. A number of methods such as statistical analysis, spectral analysis, cepstral analysis, etc. have been shown to offer high potential for the early detection of malfunctions and diagnosis of machines [1]. However, applications to internal combustion (IC) engines have remained rather limited, essentially because of the complexity of the vibration signals that are involved. Indeed, it was early realized that any effective approach dedicated to IC engines would have to cope with the highly transient nature of their vibrations. This gave little scope to the classical methods which usually assume stationarity (time invariance of statistical properties) of the vibrations and so cancel out all time information.

This paper introduces an original approach which explicitly takes into account the non-stationary nature of the vibration signals produced by IC engines. The key idea is to rephrase the prospect of vibration monitoring of IC engines into the very general framework of cyclostationary processes, which is perfectly suited for describing physical phenomena generated on a cyclic basis.

The paper is organized in two parts. On the one hand, the first part discusses the cyclostationary modelling of the vibrations issuing from IC engines and from then on, introduces a general methodology fruitful for designing new indicators dedicated to condition monitoring. On the other hand, the second part focuses on the specific issue of assessing the combustion process from vibration measurements and demonstrates how the exploitation of cyclostationarity can obviate a number of related difficulties [2].

The first part of the paper is structured as followed. In the first section, the issue of condition monitoring IC engines through vibration analysis is briefly stated. In the second

0022-460X/02/$35.00

section, it is demonstrated how the vibrations issuing from IC engines can be naturally modelled as cyclostationary stochastic processes and the advantage of using angular sampling to strengthen this property is highlighted. In the third section, a general methodology is introduced which serves as a background for designing *ad hoc* diagnostic indicators. The proposed methodology is shown to have the very important feature of being able to characterize the mechanical signatures on an angle basis locked to the kinematics of the engine. Eventually, this leads us to the definition of specific indicators called *conditional spectral moments and cumulants* which enjoy a joint angle and frequency interpretation, but other similar indicators may just as well be defined on the basis of the proposed methodology. Finally, the last section presents an application to actual vibration signals from a small diesel engine, thus demonstrating the feasibility and effectiveness of our approach.

2. CONDITION MONITORING OF IC ENGINES THROUGH VIBRATION ANALYSIS

IC engines naturally produce high levels of vibration energy when running, an effect which in most instances is undesirable since it gives rise to acoustical disturbances and may eventually lead to mechanical failures by fatigue. However, while a certain amount of vibration energy is unavoidable, this can be used profitably to give quality indicators for condition monitoring of engines. The basic idea is that every moving component or physical process involved in the operation of an engine produces a vibration signal that is uniquely its own and to which we shall refer as a *vibration signature*. It is believed that vibration signatures have the same features (at least from a statistical standpoint) for any other engine operating under the same conditions, but will differ significantly from their templates as soon as the underlying components or physical processes sustain some internal modifications. This central assumption motivates the measurement of vibration and its analysis as a means for condition monitoring.

Indeed, any successful methodology based on this idea will have to first break down the information carried by the vibration signal into its various contributions, then relate them to their respective excitation sources, and finally to issue a decision based on some *ad hoc* criteria. This versatile methodology can be applied just as well, with minor modifications, to other issues such as noise reduction or monitoring for control purposes.

The present paper is only concerned with the condition monitoring issue. First of all, it is valuable to review some of the principal malfunctions that should typically be detected through a condition monitoring procedure.

2.1. IC ENGINE MALFUNCTIONS

It is a tedious task to list all the possible malfunctions that may affect the operation of an IC engine. Nevertheless, some of them deserve special attention because of their higher criticality or higher rate of occurrence. For diesel engines, one typically looks for the presence of injection pump malfunctions, clogging or wear of the injectors, excessive clearances in the cylinders, burn out of the valves, etc. For spark ignition engines, similar malfunctions may be listed, except for the injection sub-system that is replaced by the probably more reliable ignition sub-system.

For both types of engine, the quality of the combustion process that takes place in each of the cylinders is an essential factor to achieve optimal operation. Therefore, one should make an extra effort to detect any fault that may affect it directly or indirectly. Typically, a condition monitoring procedure should be able to detect knocks in diesel engines and knocks in spark ignition engines since both phenomena can have disastrous effects if not cured in time. Incidently, the combustion process itself serves as an excellent quality indicator for condition monitoring because of its extreme sensitivity to most of the malfunctions affecting the power-train system. This prospect will be discussed in more detail in Part II of this paper [2].

2.2. VIBRATION ANALYSIS OF AN IC ENGINE: SPECIFICATIONS AND DIFFICULTIES

As stated before, every moving component or physical phenomenon taking place during operation of the engine is expected to produce a vibration signature on some point of the structure. This is to be seen schematically as a mapping from a set of events to the measured signal domain. Hence, the challenge for a condition monitoring procedure is to carry out the inverse mapping, that is to observe the excitation mechanisms from their measured effects.

Typically, the main sources of excitation that are likely to be observable from the vibration signal are associated with the following mechanisms:

- rocking and twisting of the engine block on its supports, due to the action of inertial forces,
- impacts due to clearances at links, those at the crankshaft bearings and the so-called piston slap being extremely noisy,
- closures and openings of valves,
- high-pressure injection of fuel in diesel engines,
- rapid rising of gas pressure in the cylinders during the combustion, especially in diesel engines where it has been compared with a hammer blow.

The ranking of the aforementioned excitation mechanisms according to their relative release of vibration energy is a question that is still being argued nowadays and obviously depends on many parameters. However, it is of value to know where exactly these mechanisms are likely to produce some vibration signatures in the engine cycle. Such information is usually available from the manufacturer. For instance, Figures 1 and 2 schematically display the theoretical occurrences of valve impacts, injections and piston slaps for an arbitrary 4-stroke 4-cylinder diesel engine. The amplitudes of the arrows aim at giving a very rough idea of the relative magnitudes of the expected impacting forces.

A real vibration signal will obviously differ significantly from the diagrams shown in Figures 1 and 2, because the impacting forces will not be so idealized and will actually yield some evanescent oscillations due to the dispersion and reverberation of the stress waves when propagating through the structure. Eventually, the observed signal may look considerably different from what one would expect. The impulse responses on a typical engine structure typically last for a few milliseconds: for an engine running at 1500 r.p.m., this means an angle duration greater than 10° so that within intervals where impacts occur close together the vibration signatures will inevitably overlap. This is typically what happens in the vicinity of the top dead centre, where injection, piston slap, combustion and valve impacts occur within a few degrees.

Consequently, the hard-core difficulty of vibration analysis when applied to IC engines will be to single out the contributions of each individual source over a large set of overlapping vibration signatures. Furthermore, this should be done in perfect synchronization with the engine kinematics so that the occurrence of each source can be assigned to a unique angle position in the engine cycle.

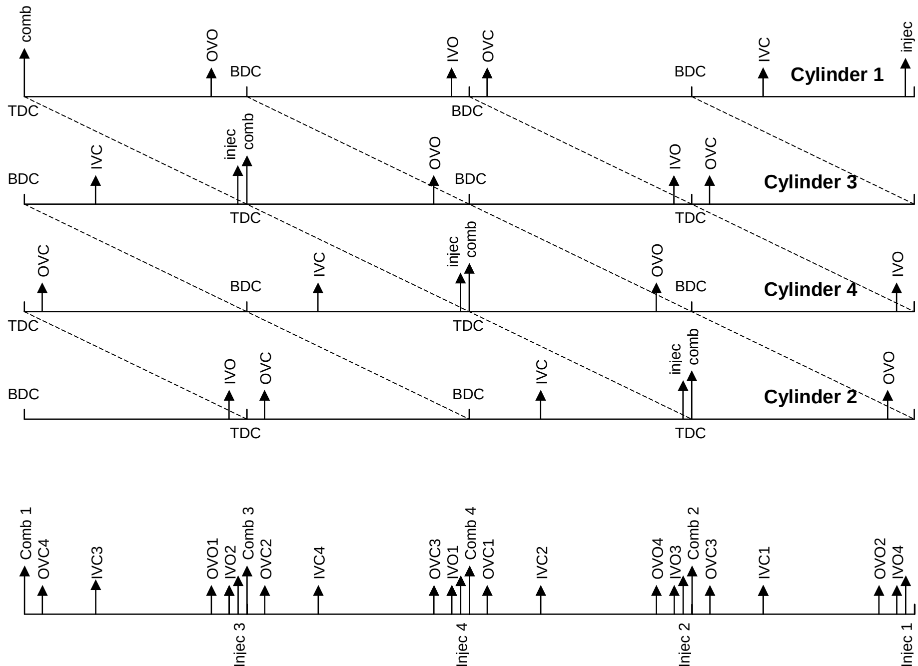

Figure 1. Theoretical occurrences of input valve opening (IVO) and closure (IVC), output valve opening (OVO) and closure (OVC), combustion and injection in each cylinder. TDC and BDC stand for top dead center and bottom dead center. The bottom diagram shows the superposition of all events within one engine cycle.

In order to achieve these goals, one has to rely on signal processing techniques that explicitly take into account the non-stationarity of the vibration signal. Even if a number of such techniques do exist, they are much more delicate to apply than the classical and well-developed techniques for stationary signals.

2.3. OVERVIEW OF EXISTING TECHNIQUES

The purpose of this section is to review some of the classical approaches that have been proposed for vibration analysis of IC engines. Their number is quite limited compared to the vast literature that exists on rotating machinery.

2.3.1. *Spectral analysis*

Spectral analysis is a direct inheritance from vibration analysis of rotating machinery. Although it has proved useful for detecting unbalance of the crankshaft or evidencing misfires, its diagnostic potential is seriously limited by its inability to locate malfunctions with respect to the engine kinematics [3]. Because it cancels out all time information, this is a technique which is only really useful under the unlikely assumption of stationarity.

2.3.2. *Envelope analysis*

This technique is inspired by the diagnosis of rolling element bearings and consists in "enveloping" the vibration signal filtered around some resonance frequencies. This processing returns an energy signature that retains time information. It has been widely used for the detection of knocks in spark ignition engines, even though it requires specific

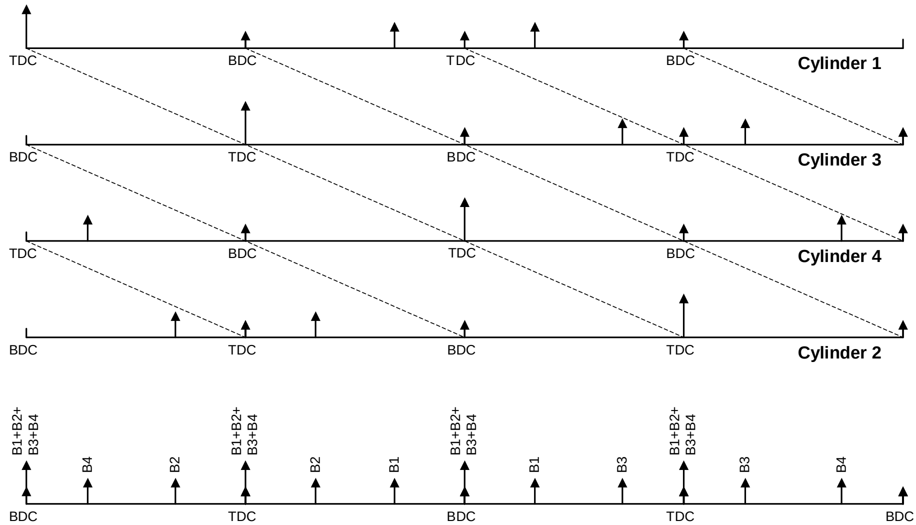

Figure 2. Theoretical occurrences of piston slap in each cylinder and superposed events within one engine cycle.

decision criteria to distinguish between its natural random fluctuations and those due to possible malfunctions [4].

2.3.3. *Time–frequency analysis*

Time–frequency analysis combines the advantages of spectral analysis and envelope analysis by conjointly characterizing the vibration signal energy in time and frequency. This makes it theoretically feasible to detect small abnormal changes in the frequency content of a vibration signature at a given time instant. The appealing potential of the technique has brought it to the fore during the last decade, but its practical use appears to be somewhat limited since the setting up and interpretation of a time–frequency representation usually requires the judgement of an expert [5–7]. Advanced studies have been undertaken on its potential application to knock detection.

2.3.4. *Other advanced techniques*

There are a number of other signal processing techniques that have been introduced to deal with some specific types of diagnosis. Probably one of the most successful pertains to Lyon [8, 9], who suggested using a deconvolution approach to reconstruct the cylinder pressure trace from its vibration signature measured on the engine structure and then to use it as a diagnostic indicator. Subsequent studies have been undertaken to refine this approach, yet it remains an active field of research as it faces serious problems of ill-posedness and robustness [10, 11]. This issue will be discussed in more detail in Part II of this paper [2].

In the following, the aim is to propose two methodologies largely inspired by the existing approaches with a major improvement due to explicit modelling of the non-stationary and stochastic nature of the vibration signals involved. The first methodology is dedicated to condition monitoring of the power-train system and generalizes envelope analysis by shortening its connection with time–frequency analysis, whereas the second one—presented in Part II of this paper—should solve a number of problems associated with the reconstruction of the pressure trace for the specific purpose of using it as a diagnostic indicator.

The central idea which motivates this work is to take advantage of the property of cyclostationarity which characterizes the vibration signals measured on IC engines.

3. CYCLOSTATIONARITY AND ANGULAR SAMPLING

Dealing with mechanical vibrations requires the simultaneous knowledge of the structure to be tested, the data acquisition system and the data processing techniques. These matters are tightly linked together and determine each other. In this section, it will be shown how efficient statistical signal processing tools can be designed from mechanical considerations and successfully applied if the appropriate acquisition mode is chosen.

The keystone is the relaxation of the stationarity assumption and its replacement by an exact modelling of the type of non-stationarity involved in IC engine vibrations. Furthermore, this is conveniently achieved by considering the vibration signals on an angular basis rather than a temporal basis, and then by exploiting the properties that this transformation offers.

3.1. CYCLOSTATIONARY PROCESSES

For a given set of constant operating parameters (speed, temperature level, load torque, indicated torque, etc.), the physical processes that generate vibrations are geared by periodic mechanisms. As a consequence, the recorded vibration signals are expected to exhibit strong periodic patterns with the same periodicity as that of the engine cycle. This is especially true for vibration signatures stemming from mechanical impacts and component motions, for which the kinematics act as a constraint on their occurrences. In practice, one also observes some non-deterministic vibrations stemming from phenomena that are less predictable, such as those related to the thermodynamics and fluid mechanics. Although these vibration signatures appear random, careful inspection of them reveals that their statistical properties are actually periodic: there are typically modulations that keep repeating identically from one cycle to another. Such processes exhibiting hidden periodicity in their randomness have been named *cyclostationary* [12]. Usually, for IC engines the cyclostationary portion of the vibration signals is found to have a much smaller power than the periodic portion, yet it should not be disregarded as it actually carries a great deal of useful information for diagnosis as will be demonstrated below.

First look in detail at what a cyclostationary process really is and what characterizes it. From the above discussion, it clearly appears that cyclostationarity is a special sub-class in the class of non-stationary stochastic processes. At least two types of cyclostationarity are to be distinguished:

(1) *first order cyclostationarity* determines processes that are periodic in their mean value, that is for a stochastic process $\{Y(t)\}\ t \in \mathbb{R}$,

$$m_Y(t) = \mathrm{E}\{Y(t)\} = m_Y(t+T), \tag{1}$$

where T is here the engine cycle period,(2)

second order cyclostationarity determines processes that are periodic in their autocovariance function, that is,

$$\begin{aligned} K_{YY}(t,\tau) &= \mathrm{E}\{Y(t-\tau/2)Y(t+\tau/2)\} - m_Y(t-\tau/2)m_Y(t+\tau/2) \\ &= K_{YY}(t+T,\tau). \end{aligned} \tag{2}$$

Note that it is important to define second order cyclostationarity with respect to the *autocovariance function* and not the *autocorrelation function* as is sometimes done: by doing so, second order cyclostationarity is clearly distinguished from first order cyclostationarity.

Therefore, the periodic portion of the vibration signal issuing from an IC engine pertains to first order cyclostationary, while its non-deterministic portion pertains to second order cyclostationarity. The same decomposition applies just as well to other types of machinery. This has been shown for gear systems where the second order cyclostationary effect is usually much weaker—if not negligible—than first order cyclostationary [13]. For rolling element bearings, it has been demonstrated that the relative contribution of the two types of cyclostationarity reverses with frequency, the vibration signal becoming purely second order cyclostationary with increasing frequencies [14].

Cyclostationary processes have a number of nice mathematical properties arising from equations (1) and (2). Interested readers are referred to references [12, 15]. However for the scope of this paper, it is sufficient to remember that for a so-called wide sense cyclostationary process, both its mean and autocovariance functions are periodic functions.

3.2. A STOCHASTIC MODEL FOR IC ENGINE VIBRATIONS

Based on physical observations, a stochastic model is proposed for the vibration signal issuing from an IC engine. Let $Y(t)$ be the observed vibration stochastic process,† $p(t)$ its deterministic part and $X(t)$ its non-deterministic part accounting for the random fluctuations around $p(t)$. Also consider an additional term $N(t)$ that accounts for some stationary background noise. Thus,

$$Y(t) = p(t) + X(t) + N(t) \tag{3}$$

with

$$\mathrm{E}\{Y(t)\} = p(t). \tag{4}$$

Assume that the engine cycle has a non-random period T. Then clearly $p(t)$ is first order cyclostationary:

$$p(t) = \mathrm{E}\{p(t)\} = p(t+T). \tag{5}$$

Without loss of generality, $X(t)$ is set to be second order cyclostationary only:

$$K_{XX}(t,\tau) = K_{XX}(t+T,\tau). \tag{6}$$

As for the additive noise, its stationarity implies

$$K_{NN}(t,\tau) = K_{NN}(\tau). \tag{7}$$

In order to single out the contributions of each part, it is necessary to impose that $X(t)$ and $N(t)$ are uncorrelated processes, i.e.,

$$K_{XN}(t,\tau) = 0. \tag{8}$$

We claim that the set of conditions (4)–(8) is enough to model the vibration signal in a unique form given by equation (3). Note that this model has already been used without

†The usual notation is adopted where stochastic processes are denoted by capital letters and deterministic functions by lower case letters.

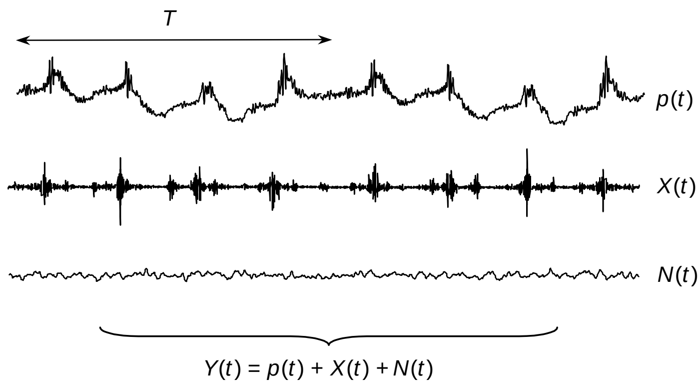

Figure 3. Stochastic model for IC engine vibrations.

explicit mention to cyclostationarity in reference [16]. Also its uniqueness relies on the presence of a unique non-random period $T > 0$. Whereas this assumption holds true for IC engines, the more general case would include a set of different cycle periods, so that there would be as many decompositions of the form (3) as there are different cycles. This is typically what would happen for example in gearboxes. Model (3) is illustrated in Figure 3.

Finally, it is important to stress that because the contributions $p(t)$ and $X(t)$ have very different statistical properties, it is compulsory to process them independently with the respective tools that suit each of them. The authors are convinced that a number of previous studies concerned with cyclostationary vibration signals did not benefit from the success they deserved because of not doing so.

3.3. DATA ACQUISITION PRINCIPLE

It can be shown that the proposed stochastic model for IC engine vibrations still holds true if the assumption of constant operating parameters is relaxed and replaced by the more realistic assumption of *statistically stationary* operating parameters. However, such a situation makes data processing more difficult since the cycle period has to be changed to its mean value $\mathrm{E}\{T\}$, a quantity that must be estimated. Moreover, one should also keep in mind that the objective is to relate vibration signatures with the engine kinematics, that is with an angle position inside the engine cycle. This is consistent with the fact that the underlying physical periodicity really belongs to an angle and not a time dimension.

For all these reasons, it was decided to sample the signals with a *constant angle period* Θ_e instead of *a time period* T_e. This can be technically accomplished by means of an encoder mounted on the engine crankshaft, which directly delivers the clock signal to use for angular sampling of the measured signals. The major contribution of angular sampling is that it makes the cycle length constant and independent of the engine speed variations. However, it requires the clock signal to deliver enough pulses per revolution so that speed variations can be effectively tracked inside the engine cycle. Examples of angular sampling applied to condition monitoring of rotating machinery can be found in references [7, 8].

From now on, the equivalent angular form of equation (3) should be rewritten as

$$Y(\theta) = p(\theta) + X(\theta) + N(\theta) \tag{9}$$

with time period T replaced by its angle counterpart Θ. For instance, for a 2 stroke engine, one has $\Theta = 2\pi$, while for a 4-stroke engine $\Theta = 4\pi$.

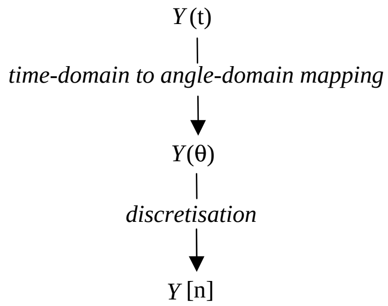

Figure 4. Data acquisition principle.

After discretization due to the sampling operation, equation (9) becomes

$$Y[n] = p[n] + X[n] + N[n], \tag{10}$$

where $Y[n]$ relates to the angle sampled version of $Y(\theta)$ at angles $\theta = n\Theta_e$ and where the angle period Θ of the engine cycle now spans an integer number N of samples so that $\Theta = N\Theta_e$. These transformations are summarized in Figure 4.

The data acquisition principle discussed in this section will next be shown to considerably simplify the data processing task and to offer new potential to the estimation issue.

4. CYCLIC PROCESSING OF VIBRATION SIGNATURES

Angular sampling is perfectly suitable for performing statistical signal processing on the engine cycle basis, that is with an exact synchronization on the engine kinematics. From a theoretical standpoint, it gives access to the easy estimation of cyclic statistics that can be designed under the assumption of cyclostationarity. This section covers these matters in detail.

4.1. CYCLIC STATISTICS WITH WOLD'S ISOMORPHISM

In order to compute statistics on a single trajectory $y[n]$ obtained from some measurement of the stochastic process $\{Y[n]\}$, $n \in \mathbb{Z}$, a natural solution consists in substituting theoretical ensemble averages by averages over the engine cycles. For discretized signals, this is well formalized through *Wold's isomorphism* [19]. The basic idea consists in building up a new stochastic process $\{Y_N[m]\}$, $m = 0, \ldots, N-1$ within the engine cycle by splitting up measurement $y[n]$ into a collection of N long sequences, with N the number of samples per cycle. Formally,

$$Y_N[m] = y[m + \zeta N], \quad m = 0, \ldots, N-1, \tag{11}$$

where ζ stands for a random point process taking integer values. The principle of the construction of $Y_N[m]$ is illustrated in Figure 5.

From now on, it is advisable to work solely on the $Y_N[m]$ process defined on the engine cycle, yet keeping in mind that the isomorphism guarantees similar statistical properties for $Y_N[m]$ and $Y[n]$:

$$m_{Y_N}[m] = m_Y[m], \tag{12}$$

$$K_{Y_N Y_N}[m, \tau] = K_{YY}[m, \tau], \quad m = 0, \ldots, N-1, \ \tau \in \mathbb{Z}. \tag{13}$$

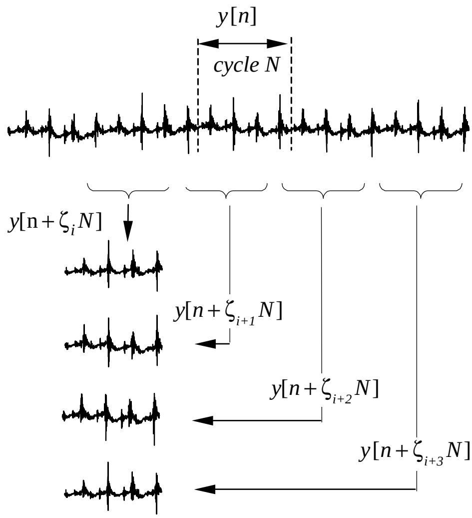

Figure 5. Wold's isomorphism principle.

The mathematical trick behind Wold's isomorphism provides consistent estimates to be defined. Indeed, it is now clear that a trivial estimator for the expected value of any arbitrary function $g[m] = g(Y[m])$ of the stochastic process $Y[m]$ is given by

$$\langle g[m]\rangle_N^I = \frac{1}{I}\sum_{i=0}^{I-1} g(y[m+iN]), \quad m = 0, \ldots, N-1, \tag{14}$$

where $\langle|\cdot|\rangle_N^I$ means averaging over I cycles of length N samples of a single measurement $y[n]$.

For instance, when applied to the estimation of the periodic component $p[n]$ in equation (10), this estimator leads to the well-known synchronous average:

$$\hat{p}[m] = \langle p[m]\rangle_N^I, \quad m = 0, \ldots, N-1. \tag{15}$$

From a theoretical standpoint, Wold's isomorphism gives to cyclostationary processes a similar completeness that the concept of ergodicity gives to stationary processes. As a matter of fact, it could be just as well summarized by the idea of cycloergodicity [12].

4.2. A CYCLIC ANGLE--FREQUENCY APPROACH

Which statistical signal processing tools should be used or designed to cope with the highly transient signals issuing from an IC engine? Because non-stationarity is not strictly speaking a property, no versatile techniques seem to exist for analyzing it. However, in early references [20, 21] the advantages of using a synchronous time–frequency analysis were clearly demonstrated. By that time, focus was put on the use of the short-time Fourier transformation. Based on the recent cyclostationarity framework, we suggest that the Wigner–Ville transformation is a good and natural alternative to the short-time Fourier transformation [22]. Indeed, several previous studies have pointed out the benefits of using the Wigner–Ville distribution among a large variety of non-stationary oriented techniques [5, 6].

The Wigner–Ville distribution provides a convenient tool for picturing patterns in the time (angle)–frequency plane, as it gives a frequency distribution of the signal at every time (angle) instants. Its success lies in the fact that it has a sound mathematical basis and appealing physical properties, among which we shall retain for upcoming purposes that [23]:

(1) the time (angle) marginal distribution gives the instantaneous power of the analyzed signal,
(2) the centre of gravity of the Wigner–Ville distribution gives the instantaneous frequency of the analyzed signal.

Most important, it has been shown that many of the existing time–frequency distributions can be derived from the Wigner–Ville distribution by passing it through a parametrization kernel.

When applied in the angle domain, the Wigner–Ville distribution of a deterministic signal $z(\theta)$ is given by

$$W_Z(\theta, \nu) = \int_{\mathbb{R}} z(\theta - \tau/2) z(\theta + \tau/2) \exp(-\mathrm{j}2\pi\nu\tau)\,\mathrm{d}\tau, \quad \mathrm{j}^2 = -1. \tag{16}$$

On the other hand, the Wigner–Ville distribution is known to have the major drawback of yielding interferences some of which have negative values, therefore making it difficult to view it as an energy spectrum. These interferences are usually reduced with a suitable smoothing kernel on $W_z(\theta, \nu)$, but at the expense of a loss in the time (angle)–frequency resolution.

Other time (angle)–frequency distributions exist which are always positive (short-time Fourier transform, Priestley's evolutive spectrum), but unfortunately none of them enjoys the same appealing physical properties as does the Wigner–Ville distribution. Generally speaking, among all time (angle)–frequency distributions, there is always a trade-off between a positive energy spectrum and the preservation of the marginal properties.

Whilst this is true for deterministic signals, we claim that this problem can be overcome by taking the expected value of the Wigner–Ville distribution on the non-deterministic contribution $X(\theta)$ in equation (9). This defines the Wigner–Ville spectrum as [24]:

$$W_{XX}(\theta, \nu) = \int_{\mathbb{R}} K_{X_a X_a}(\theta, \tau) \exp(-\mathrm{j}2\pi\nu\tau)\,\mathrm{d}\tau, \tag{17}$$

where (following Ville) the use of the analytic version $X_a(\theta)$ of $X(\theta)$ is a first step towards the reduction of interference between positive and negative frequency components.

From the cyclostationarity of $X(\theta)$,

$$W_{XX}(\theta, \nu) = W_{XX}(\theta + \Theta, \nu) \tag{18}$$

i.e., the Wigner–Ville spectrum is a periodic function and needs only being analyzed on the engine cycle basis.

This property is the cornerstone of a consistent estimator to be defined from a single trajectory of the sampled process $X[n]$:

$$\hat{W}_{XX}[m, k] = \sum_{\tau=-M}^{M} \hat{K}_{X_a X_a}[m, 2\tau] \exp\left(-\frac{\mathrm{j}4\pi\tau k}{2M+1}\right), \quad m = 0, \ldots, N-1, \tag{19}$$

where M is to be set sufficiently large and where $\hat{K}_{X_a X_a}[m, 2\tau]$ is obtained by setting

$$g[m] = X_a[m - \tau] X_a[m + \tau] \tag{20}$$

in equation (14).

The effectiveness of the proposed approach was verified on a real signal recorded on a 4 cylinder 1905 cm^3 diesel engine. The accelerometer was mounted on the middle of the engine block, in the vertical direction, and an optical shaft-encoder was placed on the end

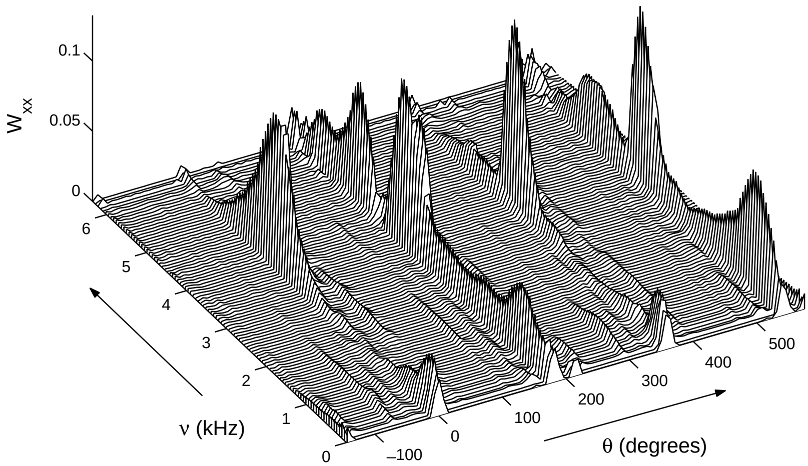

Figure 6. Averaged Wigner–Ville spectrum on 128 cycles. Computation performed on the residual signal, after extraction of the synchronous average.

of the crankshaft to give 512 pulses per revolution. With this device, the signal could be sampled directly with respect to the shaft angle without the need of post-processing the data. $I = 128$ cycles were collected for an engine speed of 1500 r.p.m. The Wigner–Ville spectrum was then computed by averaging over the $I = 128$ cycles, after extraction of the synchronous average $\hat{p}[m]$. Figure 6 displays it on the engine cycle basis scaled in degrees with 0° referencing the top dead centre of cylinder No. 1 For sake of simplicity, frequencies are scaled in Hz rather than deg^{-1}, which makes sense as long as a fairly constant engine speed is assumed.

Analysis of the Wigner–Ville spectrum reveals the following features:

(1) Among all the angle–frequency signatures that one may expect to observe in the engine cycle, those relating to the release of energy during combustion are the most pronounced. They occur at angle positions 0°, 180°, 360° and 540° for cylinders No. 1, No. 3, No. 4, and No. 2 in the firing order, and excite a resonance of the system around 4.5 kHz.
(2) In the vicinity of the combustion events, small transients are noticeable. They can be checked as relating to valve operation (openings and closures of inlet and exhaust valves).
(3) The angle–frequency patterns separated by 180° are paired because they stem from symmetrical sources with respect to the location of the accelerometer.

Now, let us investigate what the same spectrum looks like when computed on the raw vibration signal, without bothering about first extracting its periodic portion. Figure 7 shows clearly that a lot of interferences are then produced which make the interpretation of the Wigner–Ville spectrum difficult. Actually, it is an easy matter to prove that the interferences theoretically cancel out to zero if the vibration signatures in the signal are mutually statistically uncorrelated, an assumption which obviously requires the deterministic portion to be subtracted. This requirement is fully consistent with the fact that the first order and second order cyclostationary contributions in model (3) should be processed separately, with different tools. Here, the Wigner–Ville spectrum is only applicable to the latter, while an angle–frequency analysis of the former would instead necessitate the Wigner–Ville distribution to being used along with some smoothing kernel.

Finally, let us investigate the potential of the Wigner–Ville spectrum computed on the second order cyclostationary portion of the vibration signal as a tool for diagnosis. For

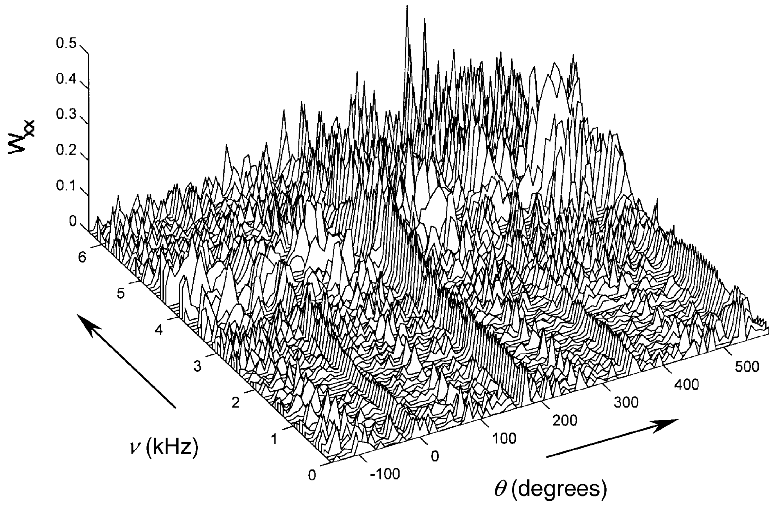

Figure 7. Averaged Wigner–Ville spectrum on 128 cycles. Computation performed on the raw signal.

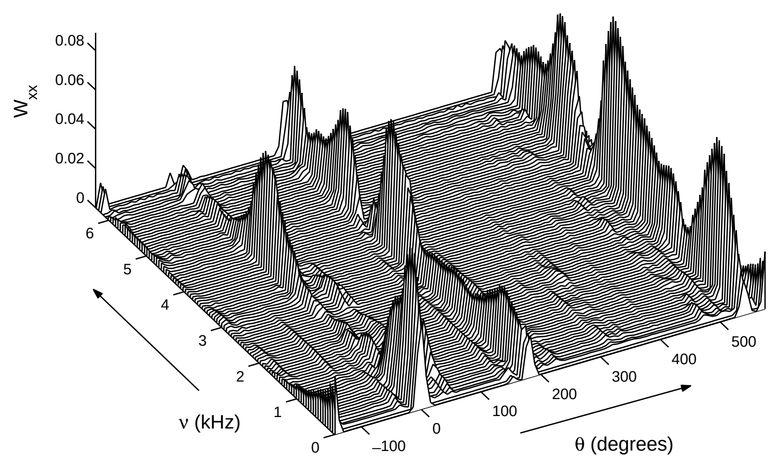

Figure 8. Wigner–Ville spectrum in case of misfires in cylinder N°4 (firing order: 1–3–4–2).

instance, Figures 8 and 9 display it after firstly simulating misfires in one of the cylinders and secondly setting a small advance of a few degrees to injection times. By comparing these spectra with that of Figure 6, it can be observed that:

- the presence of misfires in the fourth cylinder (360° in the firing order) is clearly evidenced, thus inducing a straightforward diagnosis much more evident than by solely inspecting the raw vibration signal,
- the slight advance is hardly noticeable due to the orientation of the Wigner–Ville spectrum in the 3-D space, and would require displaying it from different view angles.

These examples aimed at showing that the Wigner–Ville spectrum is an effective tool for characterizing the vibration signatures of an IC engine. However, it might sometimes be troublesome to handle for non-specialists because of its 3-D nature. Therefore, it would be of great value to summarize it on a reduced set of 2-D indicators that could just as well serve the purpose of diagnosis.

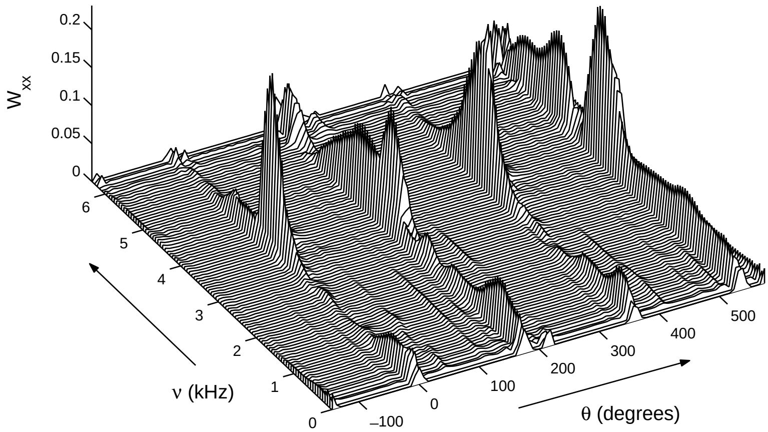

Figure 9. Wigner–Ville spectrum in case of a small injection advance.

4.3. CONDITIONAL SPECTRAL MOMENT ANALYSIS

Our approach is motivated by the similarity between the Wigner–Ville spectrum and a probability density function. Actually, under the assumption of non-negativity, the Wigner–Ville spectrum may be viewed as the probability density function of the frequency random variable ν conditioned to variable θ. This is nothing more than interpreting it as a family of juxtaposed energy spectral densities, one at each angle position. It is a classical result from probability theory that density functions can be well described by their first few moments, or even better by their first few cumulants. The former are generated by the characteristic function of the first kind of the random variable, whereas the latter are generated by its logarithmic version, i.e., the so-called characteristic function of the second kind. We claim that the Wigner–Ville "density" can just as well be summarized by its first "spectral" moments and cumulants, while some of them can also be given physical meanings.

From the above discussion, the nth spectral moment is defined as

$$m_n^X(\theta) = \int_0^\infty W_{XX}(\theta, \nu)\nu^n \,\mathrm{d}\nu. \tag{21}$$

It is proven in reference [25] that spectral moments can be found from a generating function which actually happens to be the autocovariance function of the analyzed stochastic process:

$$m_n^X(\theta) = (2\pi \mathrm{j})^{-n} \frac{\partial^n K_{XX}}{(\partial\tau)^n}(\theta, 0). \tag{22}$$

Extending this result, it is possible to define the spectral cumulants as

$$\kappa_n^X(\theta) = (2\pi \mathrm{j})^{-n} \frac{\partial^n \ln K_{XX}}{(\partial\tau)^n}(\theta, 0). \tag{23}$$

Spectral cumulants have one advantage over spectral moments for they can be given physical meanings (at least the first few of them). To illustrate this point, consider a dynamic structure one of whose modes is stochastically excited by a delta correlated force. Figure 10(a) shows the schematic Wigner–Ville spectrum of the structural response with possible slight variations with respect to angle. Figure 10(b) displays a slice of the Wigner–

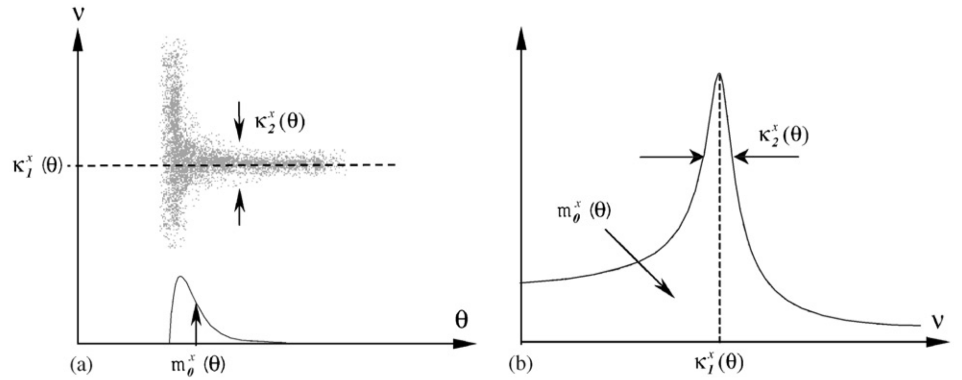

Figure 10. (a) The Wigner–Ville spectrum viewed as the probability density function of variable ν conditioned to variable θ. (b) Physical interpretation of the spectral moments and cumulants.

Ville spectrum taken along the frequency axis for a given angle, which we shall refer to as the energy spectral density at angle θ.

From probability theory one knows that

- $m_0^X(\theta)$ is the area under the energy spectral density, i.e., the *mean instantaneous vibration power* dissipated by the mode,
- $\kappa_1^X(\theta)$ is the centre of gravity of the energy spectral density, i.e., a measure of the *mean instantaneous natural frequency* of the mode,
- $\kappa_2^X(\theta)$ is the radius of gyration of the energy spectral density, i.e., a measure of the *mean instantaneous damping* of the mode,
- $\kappa_3^X(\theta)$ and $\kappa_4^X(\theta)$ may be further viewed as *mean instantaneous measures of skewness and flatness* of the mode.

These results are truly valid when only one mode of the structure is predominant over the others, if any. In general, when some other modes arise in the structural response, the former modal interpretations hardly hold any more, even though $m_0^X(\theta)$, $\kappa_1^X(\theta)$ and $\kappa_2^X(\theta)$ can still be interpreted as *instantaneous power*, *instantaneous central frequency and instantaneous bandwidth* of the process under analysis. From this point of view, $m_0^X(\theta)$ and $\kappa_1^X(\theta)$ actually achieve a stochastic amplitude and frequency demodulation.

In accordance with the principle of data reduction, we propose compressing the Wigner–Ville spectrum information onto a set of a few monodimensional sequences given by the spectral moments and cumulants. This can be achieved with fast estimation procedures directly from the angular sampled signal without the need to first compute the Wigner–Ville spectrum.

The formulae are given below for the first sampled spectral moments [25]:‡

$$\hat{m}_0^X[m] = \langle |X_a[m]|^2 \rangle_N^I, \quad m = 0, \ldots, N-1, \tag{24}$$

$$\hat{m}_1^X[m] = \frac{1}{2\pi} \operatorname{Im} \langle X_a^{(1)}[m] X_a[m] \rangle_N^I, \tag{25}$$

$$\hat{m}_2^X[m] = \frac{1}{8\pi^2} \operatorname{Re} \langle |X_a^{(1)}[m]|^2 - X_a^{(2)}[m] X_a[m] \rangle_N^I, \tag{26}$$

‡ $X_a^{(n)}[m]$ denotes the discretised version of the nth derivative of the analytic process $X_a(\theta)$.

$$\hat{m}_3^X[m] = \frac{-1}{32\pi^3}\,\mathrm{Im}\langle X_a^{(3)}[m]X_a[m] + 3X_a^{(1)}[m]X_a^{(2)}[m]\rangle_N^I, \tag{27}$$

$$\hat{m}_4^X[m] = \frac{1}{128\pi^4}\,\mathrm{Re}\langle X_a^{(4)}[m]X_a[m] - 4X_a^{(3)}[m]X_a^{(1)}[m] + 3|X_a^{(2)}[m]|^2\rangle_N^I \tag{28}$$

from which the sampled spectral cumulants can be deduced:

$$\hat{\kappa}_1^X[m] = \frac{\hat{m}_1^X[m]}{\hat{m}_0^X[m]}, \quad m = 0, \ldots, N-1, \tag{29}$$

$$\hat{\kappa}_2^X[m] = \frac{\hat{m}_2^X[m]}{\hat{m}_0^X[m]} - \hat{\kappa}_1^X[m]^2, \tag{30}$$

$$\hat{\kappa}_3^X[m] = \frac{\hat{m}_3^X[m]}{\hat{m}_0^X[m]} - 3\hat{\kappa}_1^X[m]\frac{\hat{m}_2^X[m]}{\hat{m}_0^X[m]} + 2\hat{\kappa}_1^X[m]^3, \tag{31}$$

$$\hat{\kappa}_4^X[m] = \frac{\hat{m}_4^X[m]}{\hat{m}_0^X[m]} - 4\hat{\kappa}_1^X[m]\frac{\hat{m}_3^X[m]}{\hat{m}_0^X[m]} + 12\hat{\kappa}_1^X[m]^2\frac{\hat{m}_2^X[m]}{\hat{m}_0^X[m]} - 3\left(\frac{\hat{m}_2^X[m]}{\hat{m}_0^X[m]}\right)^2 - 6\hat{\kappa}_1^X[m]^4. \tag{32}$$

At this stage, it should be realized that computing the spectral moments and cumulants directly from the vibration signal is a decisive argument in favour of their use. Not only does it avoid the tedious task of computing the Wigner–Ville spectrum, but it also returns unbiased results: Formulae (24)–(28) and (29)–(32) are exact in the sense that they are directly deduced from definitions (22) and (23) on continuous quantities, whereas a direct integration over the estimated Wigner–Ville spectrum could only be done numerically and then approximately from discretized quantities.

The first spectral moment and first four spectral cumulants were estimated from $I = 128$ cycles of the same data as used previously, thus providing comparisons with the Wigner–Ville spectrum of Figure 6.

Figure 11 displays, respectively, the estimated sampled mean instantaneous power $\hat{m}_0^X[m]$, frequency $\hat{\kappa}_1^X[m]$, bandwidth $\hat{\kappa}_2^X[m]$, skewness $\hat{\kappa}_3^X[m]$ and flatness $\hat{\kappa}_4^X[m]$.

Analysis of these figures reveal the following points:

(1) The mean instantaneous power clearly shows the high release of power resulting from each combustion near the top-dead-centres, just as the Wigner–Ville spectrum did.
(2) The mean instantaneous frequency which is a normalized quantity does not depend on the power of the vibration signatures, but only on their frequency distributions. It exhibits sharp peaks whenever a source excites some high resonance of the structure.
(3) The mean instantaneous bandwidth is very sensitive to short transients in the vibration signals, that is vibration signatures with large frequency distributions. Alternatively, this also explains why it is less dominated by the combustion occurrences (with shorter spectral contents) than the former indicators.
(4) The mean instantaneous skewness and flatness exhibit a lot of peaks but at the same time are more difficult to interpret. For the tested engine, it was attractive that combustion onsets could be detected on these two indicators by searching for their negative values, but this determination could not be verified on some other type of engines.

Finally, after identifying all the peaks (in particular in the mean instantaneous frequency $\hat{\kappa}_1^X[m]$ and bandwidth $\hat{\kappa}_2^X[m]$), the kinematics diagram in Table 1 was established and compared with its theoretical counterpart given by the manufacturer. Small differences could be observed, which are likely to be explained by the propagation times

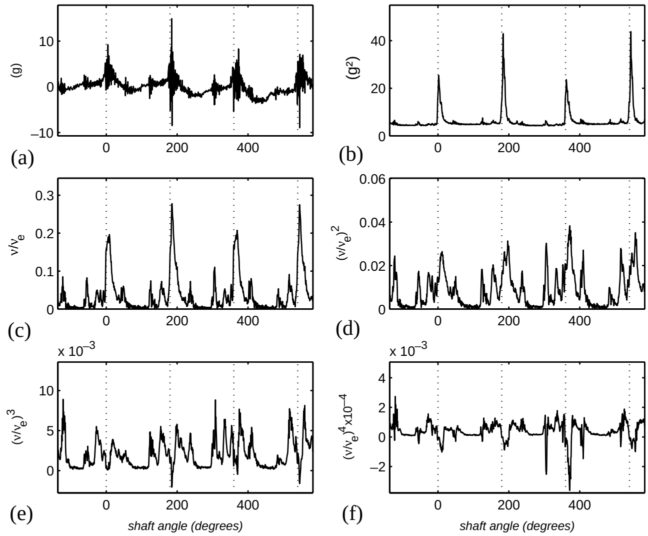

Figure 11. Estimated spectral moments and cumulants on 128 cycles. In reading order: (a) synchronous average $\hat{p}[m]$, (b) mean instantaneous power $\hat{m}_0^x[m]$, (c) mean instantaneous frequency $\hat{\kappa}_1^x[m]$, (d) mean instantaneous bandwidth $\hat{\kappa}_2^x[m]$, (e) mean instantaneous skewness $\hat{\kappa}_3^x[m]$, (f) mean instantaneous kurtosis $\hat{\kappa}_4^x[m]$.

TABLE 1

Kinematics diagram

Events	Theoretical positions	Measured positions
Inlet valve opening	$-19.5°$	$-20°$
Exhaust valve opening	$-55.5°$	$-61°$
Inlet valve closure	$49.5°$	$53°$
Nozzle needle opening	$\sim -7°$	$10°$
Piston slaps	$77.0°$ and $-43.5°$	$88°$ and $-43°$

that have not been taken into account and possibly some advanced wear of the tested engine.

However, it is important to realize that similar results would have been impossible to get from the raw signal or from its synchronous average. Figure 12 illustrates this point by simultaneously comparing the theoretical kinematic diagram with the mean instantaneous frequency $\hat{\kappa}_1^X[m]$ and the synchronous average $\hat{p}[m]$.

5. APPLICATION TO CONDITION MONITORING

5.1. A CONDITION MONITORING PROCEDURE

Before presenting examples of applications of the proposed indicators on actual industrial cases, it is necessary to decide how and when to use them in a condition monitoring procedure. In fact, it should not be forgotten that the Wigner–Ville spectrum and its spectral moments and cumulants are built on the second order cyclostationary portion of the vibration signal which usually only accounts for a small proportion of the global vibration energy in the signal.

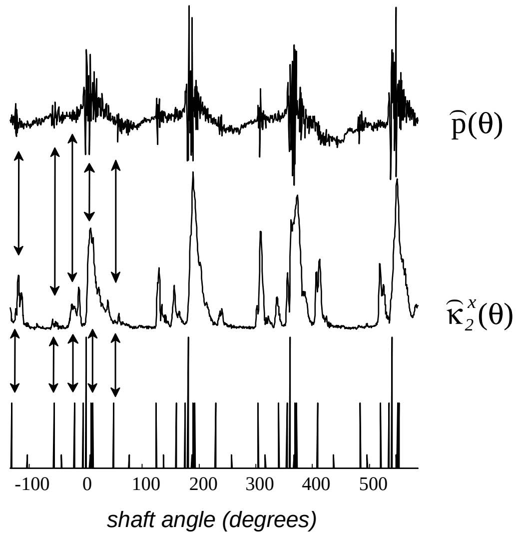

Figure 12. Comparison of the theoretical kinematic diagram with the synchronous average and the mean instantaneous frequency.

Therefore, the strong periodic part of the vibration signal must be first analyzed in the condition monitoring procedure. Then the proposed angle–frequency indicators should come at a second stage to refine the analysis. From our experience, it was found very convenient to split the condition monitoring procedure into two levels.

5.1.1. 1*st level*

The first level is fed with the synchronous average $\hat{p}[m]$ which estimates the periodic contribution to the signal and possibly with the first spectral moment $\hat{m}_0^X[m]$ which estimates the mean instantaneous power of the second order cyclostationary contribution. These two indicators have proved to be very stable for normal operating conditions, while they are usually sufficient to detect evidence of malfunctions. The underlying idea of the procedure is explained shortly hereafter. On the one hand, for a normally operating engine, all the excitation mechanisms should be identical from one cylinder to another. Therefore the resulting vibration signal is expected to have a periodic part and an instantaneous power release that are periodic with the firing (despite some differences in the signal path), i.e., with period $p\pi/N_c$ for a p-stroke and N_c-cylinder engine. On the other hand, for a defective engine, some unbalance in the excitation mechanism would yield a periodicity on the engine cycle, i.e., $p\pi$.

It is shown in reference [25] that many different faulty configurations can be detected by analyzing the relative magnitude of the Fourier coefficients of the synchronous average and mean instantaneous power. The detection process can be easily automated by means of statistical tests or any other pattern recognition technique. In the case of an alarm, one then goes to the second level of the condition monitoring procedure.

5.1.2. 2*nd level*

The second level is fed with the synchronous average, the first spectral moment and cumulant and optionally some higher order cumulants. Their analysis is done visually

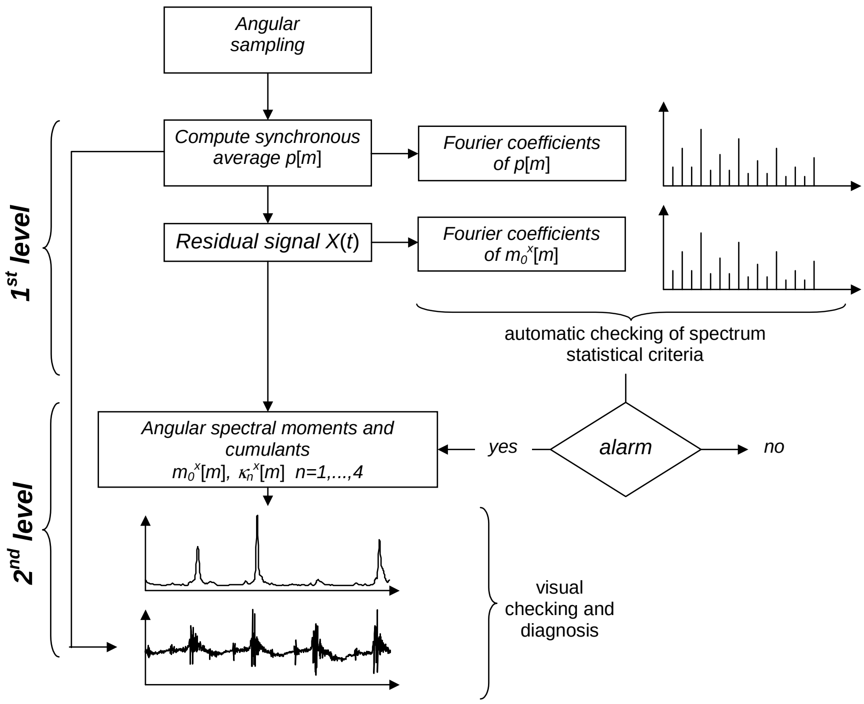

Figure 13. Principle of the condition monitoring procedure.

rather than automatically in order to localize the malfunction in the engine cycle and ultimately to identify it.

The principle of the two level condition monitoring procedure is illustrated in Figure 13.

5.2. EXAMPLE OF MALFUNCTION DIAGNOSIS

The proposed condition monitoring procedure was applied to real industrial cases. For the sake of completeness, examples are given for the same diesel engine whose data have been used up to now. The engine was actually known to have a faulty injection pump. The type of induced malfunctions was not so clear, but for some configurations of the pump, it was possible either to yield misfires or knocks although we had no way of quantifying them. Furthermore, by slightly turning the injection pump in one way or another, it was possible to get either advance or delay. Table 2 summarizes the four experiments which were run.

For each experiment, a statistical test was performed on the Fourier coefficients of the synchronous average and mean instantaneous power according to the procedure described above. The alarm was triggered for cases 2, 3 and 4, but not for case 1 because the advance was equally distributed over all the cylinders so that no unbalance could be automatically detected among the four cylinders. Actually, in order to detect equally distributed timing shifts, one should directly go to level two of the proposed procedure. The reason why the alarm was triggered for case 2 is explained below.

Figures 14 and 15 display the cyclic indicators computed for Experiments $N^\circ 1$ to $N^\circ 4$ and overlaid with those of Experiment $N^\circ 0$ which provides the reference templates. Here only the mean instantaneous power and the mean instantaneous frequency are displayed

TABLE 2

PSA XUD9 diesel engine

	Experiments
0	Normal operation
1	Advance of injections
2	Excessive delay of injections
3	Induced misfires
4	Induced knocks

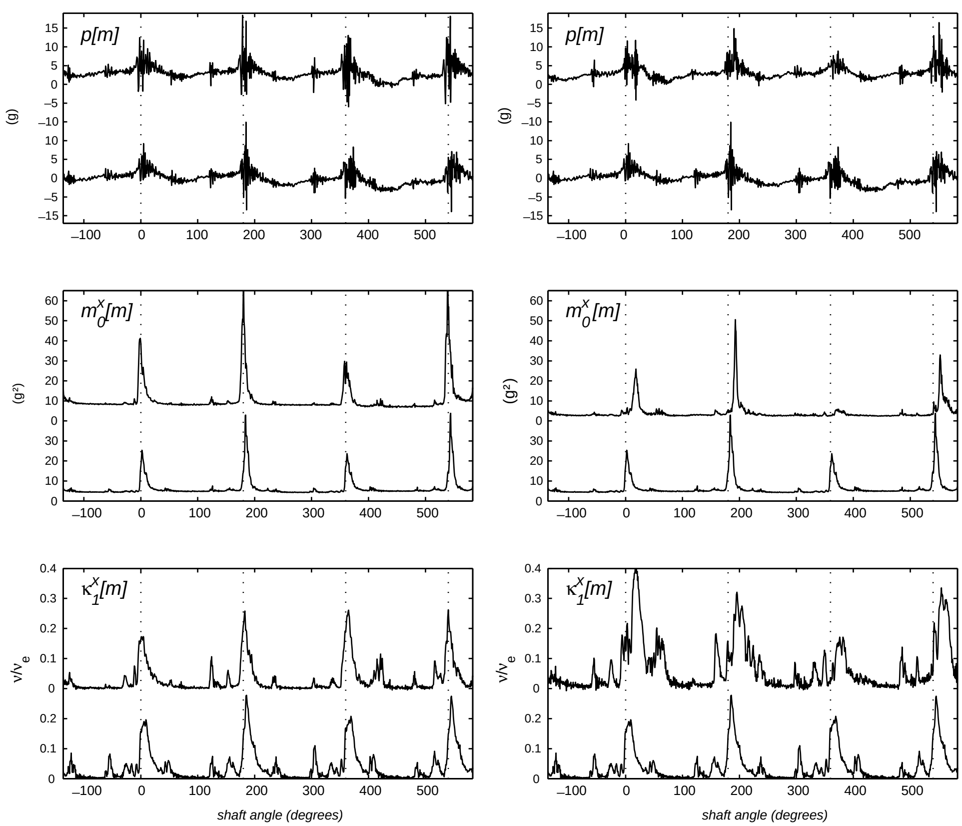

Figure 14. Estimated spectral moments and cumulants in case of injection advance (left panels) and injection delay (right panels). From top to bottom: synchronous average, mean instantaneous power and mean instantaneous frequency. In each panel, the lower plot displays the template indicator.

along with the synchronous average, as it was found that they were enough to achieve practical diagnosis. Analysis of these figures pertains to the second level of the condition monitoring procedure.

5.2.1. *Timing measurements for advances and delays*

Advance and delay were clearly detectable on any of the spectral moment or cumulant indicators by checking the position of the sudden rises due to combustion in each cylinder

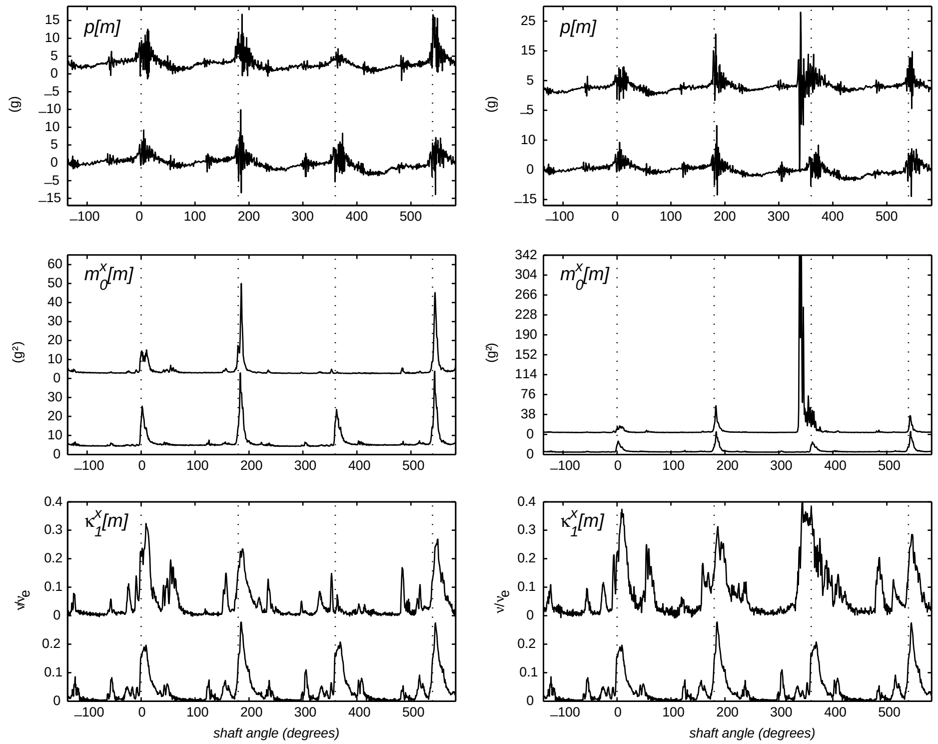

Figure 15. Estimated spectral moments and cumulants in case of misfires (left panels) and knocks (right panels). From top to bottom: synchronous average, mean instantaneous power and mean instantaneous frequency. In each panel, the lower plot displays the template indicator.

around shaft angles 0, 180, 360 and 540°. The advance was measured to be approximately 2° and the delay to be 8° with respect to the TDC. Getting such a precision from the raw vibration signal would have been quite impossible.

It was observed that the excessive delay entailed some dramatic effects in the fourth cylinder around angle 360°, very likely due to incomplete combustion in that cylinder.

5.2.2. *Detection of knocks and misfires*

Misfires and knocks were easily detectable in the fourth cylinder. The diagnosis of knocks was straightforward and no misinterpretation was possible: the mean instantaneous power reaches 310 g^2 compared to 20 g^2 in normal condition, that is a difference of 12 dB. It should be pointed out that the spectral moment in Figure 15 is very similar to that in Figure 14. However, misfire was induced by an excessive retard in the former while it was due to a pump malfunction in the latter. This illustrates the inherent difficulty in diagnosis to infer the cause of a malfunction by observing only its effects.

6. CONCLUSION

Whereas vibration-based condition monitoring of rotating machinery has become nowadays a well-mastered discipline with its own specific tools, no such ascertainment seems to exist for IC engines. This is mainly due to the delicate nature of the vibration signals to be processed which turn out to be strongly non-stationary. The aim of this paper was to show that nevertheless, vibration analysis can be applied to IC engines and sound results obtained as long as an adequate methodology is used. The solution lies in bypassing the classical assumption of stationarity or quasi-stationarity by explicitly modelling the type of non-stationarity involved: this was achieved through the paradigm of cyclostationarity.

Within the cyclostationary framework, some basic principles were then introduced that make it possible to characterize the vibration signatures in correspondence with the kinematics of the engine. This was found to be significantly simpler if angular sampling was used instead of time sampling. From a theoretical standpoint, angular sampling cleverly completes the paradigm of cyclostationarity, while from a practical standpoint, it really makes cyclic data processing feasible. These results were then applied to design some specific statistics dedicated to diagnosis. It was proposed to tackle this issue in the angle–frequency domain since it provides a full description for non-stationary signals. Accordingly, the Wigner–Ville spectrum appeared to be an interesting candidate, but required to be compressed onto a reduced set of 2-D indicators that could be more conveniently used for diagnosis. This led to the definition of the spectral moments and cumulants which can be given sound physical meanings and are closely related to a stochastic amplitude and frequency demodulation. Finally, the effectiveness of the spectral moments and cumulants for diagnosis was verified on actual industrial cases.

The main innovation of this paper is the proposal of a methodology that combines cyclostationary modelling of the vibration signals in parallel with their angular sampling. Without reference to cyclostationary the theoretical definition of cyclic indicators could not be carried out. Alternatively, without relying on angular sampling, their practical estimation could hardly be performed. Another important result was to provide evidence of the necessity of independently processing the periodic and the second order cyclostationary contributions of the vibration signal, a fact which has rarely been recognized in previous studies.

Finally, the proposed cyclic methodology is a framework in which a large variety of indicators may be designed. For example, the proposed approach based on the Wigner–Ville spectral moments and cumulants may just as well be replaced by a parametric angle–frequency approach that would directly yield a finite set of 2-D indicators based on the instantaneous autoregressive coefficients or the instantaneous modal parameters from which they could be deduced. The authors hope that the few principles outlined in this paper will motivate further studies on the application of vibration analysis in condition monitoring of IC engines.

REFERENCES

1. R. B. Randall 1987 *Frequency Analysis*. Copenhagen: Brüel & Kjaer.
2. J. Antoni, J. Daniere, F. Guillet and R. B. Randall 2002 *Journal of Sound and Vibration* **257**, 839–856. Effective vibration analysis of IC engines using cyclostationarity. Part II—New results on the reconstruction of the cylinder pressures.
3. V. Work 1991 *Machinery Vibration*. New York: McGraw-Hill.
4. J. S. Mitchell 1981 *Machinery Analysis and Monitoring*. Tulsa, OK: PennWell.
5. B. Swami and G. Rizzoni 1996 *Proceedings IEEE* **84**, 1330–1343. Mechanical signature analysis using time–frequency signal processing: application to IC engine knock detection.
6. M. Chiollaz and B. Favre 1990 *Traitement du Signal* **8**, 302–318. Caractérisation fine de bruit moteur par analyse temps-fréquence (in French).
7. O. Bardou and M. Sidahmed 1994 *Mechanical Systems and Signal Processing* **8**, 551–570. Early detection of leakages in the exhaust and discharge systems of reciprocating machines by vibration analysis.

8. R. H. Lyon and R. G. DeJong 1984 *Journal of Vibration, Acoustics, Stress, and Reliability in Design* **106**, 17–21. Design of a high-level diagnostic system.
9. R. H. Lyon, 1988 *Sound and Vibration*, 18–22. Vibration based diagnostics of machine transients.
10. P. Azzoni and M. Marseguerra 1995 *Mechanical Systems and Signal Processing* **9**, 119–128. Assessment of the potential of a Wiener–Hilbert filter for automatic diagnosis of spark ignition engine faults.
11. R. B. Randall, Y. Ren and H. Ngu 1996 *International Seminar on Modal Analysis* 21, *Leuven* 1996, 847–856. Diesel engine cylinder pressure reconstruction.
12. W. A. Gardner 1986 *Introduction to Random Processes*. New York: MacMillan.
13. C. Capdessus, M. Sidahmed and J.-L. Lacoume 2000 *Mechanical Systems and System Processing* **14**, 371–385. Cyclostationary processes: application in gear faults early diagnosis.
14. R. B. Randall and J. Antoni 2001 *Mechanical Systems and Signal Processing* **15**. The relationship between spectral correlation and envelope analysis in the diagnosis of bearing faults and other cyclostationary machine signals.
15. H. L. Hurd 1989 *Journal of Multivariate Analysis* **29**, 53–67. Representation of strongly harmonizable periodically correlated processes and their covariances.
16. S. Braun and B. Seth 1980 *Journal of Sound and Vibration* **70**, 513–522. Analysis of repetitive mechanism signatures.
17. E. Lopatinskaia, J. Zhu and J. Mathew 1995 *Mechanical Systems and System Processing* **9**, 647–655. Monitoring varying speed machinery vibrations—II. Recursive filters and angle domain.
18. M. El Badaoui 1999 *Ph.D. Thesis*. *University of Saint-Etienne* (*in French*) Contribution of cepstral analysis to the vibration analysis of complex gearboxes.
19. H. O. A. Wold 1948 *Ann. Math. Statist.* **19**, 558–567. On prediction in stationary time-series.
20. R. B. Randall 1979 *Technical Diagnostics*'79, *Karlovy Vary*, 159–168. Diagnostic of slow speed reciprocating machines.
21. J. Courrech 1979 *Motor Symposium*'79, *Strbske Gleso*, Vol. 2, 479–488. Contribution to monitoring of slow speed diesel engines.
22. D. König and F. Böhme 1994 *IEEE International Conference on Acoustic*, *Speech and Signal Processing*, Vol. 4, 149–152. Application of cyclostationary and time–frequency analysis to car engine diagnosis.
23. B. Picinbono 1989 *Time and Frequency Representation of Signals and Systems*. Berlin: Springer-Verlag.
24. W. Martin 1982 *International Conference on Acoustics*, *Speech and Signal Processing*, *Paris*, 1325–1328. Time–frequency analysis of random signals.
25. J. Antoni 2000 *Ph.D. Thesis*, *National Polytechnic Institute of Grenoble* (*in French*). Contribution of Angular Sampling and Cyclostationarity to the Vibration Analysis and Diagnosis of IC Engines.
26. S. Braun 1986 *Mechanical Signature Analysis*, *Theory and Applicatio*, London: Academic Press.